Students often wonder why ‘x(x-3) < 0’ doesn’t imply ‘x < 0 or (x – 3) < 0’. In this post, we will discuss why and we will see what it actually implies. Also, we will look at how we can handle such questions quickly.

When you see ‘< 0’ or ‘> 0’, read it as ‘negative’ or ‘positive’ respectively. It will help you think clearly.

So the question we are considering today is:

Question: For what values of x will x(x-3) be negative?

Solution: Before we try to answer this question, think – when will the product of 2 numbers be negative? When one and only one of the factors is negative. Therefore, either x should be negative or (x – 3) should be negative, but not both. Let’s consider each case.

Case 1: x is negative and (x – 3) is positive

x < 0

and (x – 3) > 0 which implies x > 3

This is not possible. x cannot be less than 0 and greater than 3 at the same time. Hence this case gives us no appropriate values for x.

Case 2: x is positive and (x – 3) is negative

x > 0

and (x – 3) < 0 which implies x < 3

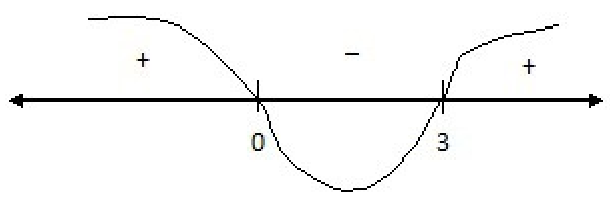
x must be greater than 0 but less than 3.

Therefore, the range of values for which this inequality will be satisfied is 0 < x < 3.

Will we have to do this every time we have multiple factors in an inequality? What will happen in case there are lots of factors? There is an easier way of handling such situations. I will first discuss the method and later explain the logic behind it.

Method: Say we have an inequality of the form (x – a)(x – b)(x – c) < 0. (For clarity, we will work with the example x(x – 3) < 0 discussed above.) This is how we solve for x:

Step 1: Make a number line and plot the points a, b and c on it. In our example, a = 0 and b = 3. The number line is divided into sections by these points. In our example, it is divided into 3 sections – greater than 3, between 0 and 3 and less than 0.



Step 2: Starting from the rightmost section, mark the sections with alternate positive and negative signs. The inequality will be positive in the sections where you have the positive signs and it will be negative in the sections where you have the negative signs.

Therefore, $x(x - 3)$ will be negative in the section $0 < x < 3$ and positive in the other two sections.

Hence, the values of x for which $x(x - 3) < 0$ is satisfied is $0 < x < 3$.

Explanation: When we plot the points on the line, the number line is divided into various sections. Values of x in the right most section will always give you positive value of the expression. The reason for this is that if $x > 3$, all factors will be positive i.e. x and $(x - 3)$, both will be positive.

When you jump to the next region i.e. between $x = 0$ and $x = 3$, the values of x will give you negative values for the entire expression because now, only one factor, $(x - 3)$, will be negative. All other factors will be positive.

When you jump to the next region on the left where $x < 0$, expression will be positive again because now both factors x and $(x - 3)$ are negative. The product of two negatives is positive so the expression will be positive again and so on…

Similarly, you can solve a question with any number of factors. Next week, we will look at how to easily handle numerous complications that can arise.